Change of Base Formula

Find x: $2^x = 8$

$x = 3$

$2^x = 10$

$\log_2 2^x = \log_2 10$

$x = \log_2 10$

change of base formula

$$\log_c a = \frac{\log a}{\log c} \text{ and } \log_c a = \frac{\ln a}{\ln c}.$$

Example: $\log_2 10 = \frac{\log 10}{\log 2}$ or $\frac{\ln 10}{\ln 2} \approx 3.322$

$\log_{12} 30 = \frac{\log 30}{\log 12} = 1.369$ ($12^? = 30$)

$\log_5 8 = \frac{\log 8}{\log 5} = 1.292$ ($5^? = 8$)

Solving Equations:

Written directions shown below are optional for the notebook check:

Solve: $9^{2x} = 27^{x-1}$

$(3^2)^{2x} = (3^3)^{x-1}$ Write with the same bases if possible.

$3^{4x} = 3^{3(x-1)}$

$4x = 3(x - 1)$ Exponents are then equal.

$4x = 3x - 3$ Solve

$x = -3$ Check

$9^{-6} = 27^{-4}$ ✓

Solve: $2^x = 5$ Not "possible" to write with the same bases.

$\log_2 2^x = \log_2 5$ Take log of each side

$x = \log_2 5$ Simplify

$x = \frac{\log 5}{\log 2} = 2.322$ Use change-of-base

Solve: $7^{9x} = 15$

$\log_7 7^{9x} = \log_7 15$ Take log of each side

$9x = \log_7 15$ Simplify

$9x = \frac{\log 15}{\log 7} = 1.392$ Use change-of-base

$x = 0.155$

New homework: Due Friday, March 14

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Let's do a few of these together:

3. $5^{x-4} = 25^{x-6}$

$5^{x-4} = (5^2)^{x-6}$

$5^{x-4} = 5^{2(x-6)}$

$x-4 = 2(x-6)$

$x-4 = 2x-12$

$8 = x$

$5^4 = 25^2$ ✓

9. $36^{5x+2} = \left(\frac{1}{6}\right)^{11-x}$

$(6^2)^{5x+2} = (6^{-1})^{11-x}$

$6^{2(5x+2)} = 6^{-1(11-x)}$

$10x+4 = -11+x$

$9x = -15$

$x = \frac{-15}{9} = \frac{-5}{3}$

12. $8^x = 20$

$\log_8 8^x = \log_8 20$

$x = \frac{\log 20}{\log 8} \approx 1.441$

13. $e^{-x} = 5$

$\ln e^{-x} = \ln 5$

$-x = 1.609$ $\quad x = -1.609$